High order corrections to density and temperature of fermions and bosons from quantum fluctuations and the CoMD-α Model

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Fragmentation experiments could provide information about nuclear matter properties and constrain the nuclear equation of state (EOS) [1-4]. Long ago, W. Bauer stressed the crucial influence of Pauli blocking on the momentum distributions of nucleons emitted in heavy ion collisions near the Fermi energy [5]. We have recently proposed a method to estimate the density and temperature based on fluctuations estimated from an event by event determination of fragment momenta and yields arising after the energetic collision [6]. A similar approach has also been applied to experimentally observe the quenching of fluctuations in a trapped Fermi gas [7] and the enhancement of fluctuations in a Bose condensate [8]. We go beyond the method of [7, 8] by including quadrupole fluctuations as well to have a direct measurement of densities and temperatures for subatomic systems (Fermions or Bosons). We recently extend the method to derive the entropy of the system and have shown how to recover the classical limit for fermions when the temperatures are large compared to the Fermi energy. Furthermore, we have also shown how to derive the density and temperature for bosons in the same scenario. We examine the collision dynamics by means of the Constrained Molecular Dynamics (CoMD) model, which allows an event-by-event analysis of the reaction mechanisms that is necessary in order to calculate fluctuations.

Following [10] a quadrupole $Q_{xy} = < p_x^2 - p_y^2 >$ is defined in a direction transverse to the beam axis (z-axis) and the average is performed, for a given particle type, over events. Such a quantity is zero in the center of mass of the equilibrated emitting source. Its variance is given by the simple formula:

$$\sigma_{xy}^2 = \int d^3p\, (p_x^2 - p_y^2)^2 n(p) \qquad (1)$$

where n(p) is the momentum distribution of particles. In [10] a classical Maxwell-Boltzmann distribution of particles at temperature T_{cl} was assumed which gives: $\sigma_{xy}^2 = \overline{N}(2mT_{cl})^2$, m is the mass of the fragment. $\overline{N}$ is the average number of particles.

In heavy ion collisions, the produced particles do not follow classical statistics, thus the correct distribution function must be used in Eq. (1). Protons(p), neutrons(n), tritium etc. follow the Fermi statistics while, deuterium, alpha particles etc., even though they are constituted of nucleons, should follow the Bose statistics.

For fermions, we will concentrate on, in particular, p and n which are abundantly produced in the collisions thus carrying important informations on the densities and temperatures reached. Using a Fermi-Dirac distribution n(p),

$$\sigma_{xy}^2 = \overline{N}(2mT)^2 F_{QC}\left(\frac{T}{\varepsilon_f}\right) \qquad (2)$$

Where $F_{QC}\left(\frac{T}{\varepsilon_f}\right) = 0.2(\frac{T}{\varepsilon_f})^{-1.71} + 1$ is the quantum correction factor which should converge to one for high T (classical limit).

Within the same framework we can calculate the fluctuations of the p, n multiplicity distributions. These are given by [11]. Since in experiments or modeling one recovers the normalized fluctuations, it is better to find a relation between the normalized temperatures as function of the normalized fluctuations. It is useful to parameterize the numerical results as:

$$\frac{T}{\varepsilon_f} = -0.442 + \frac{0.442}{(1 - \frac{\langle(\Delta N)^2\rangle}{\bar{N}})^{0.656}} + 0.345\frac{\langle(\Delta N)^2\rangle}{\bar{N}} - 0.12(\frac{\langle(\Delta N)^2\rangle}{\bar{N}})^2 \quad (3)$$

which is practically indistinguishable from the numerical result. Since from experimental data or models it is possible to extract directly the normalized fluctuations, one can easily derive the value of $\frac{T}{\varepsilon_f}$ from Eq. (3). Substuting Eq. (3) into Eq. (2), the temperature and fermi energy for the fermion system can be obtained. Then the density of the system can be extracted from

$$\varepsilon_f = 36(\frac{\rho}{\rho_0})^{\frac{2}{3}} \quad (4)$$

Where $\rho_0 = 0.16\ \mathrm{fm}^{-3}$ is the normal density. Once the density and the temperature of the system have been determined it is straightforward to derive other thermodynamical quantities. One such quantity is the entropy:

$$S = \frac{U-A}{T} \quad (5)$$

U and A are the internal and Helmotz free energy respectively[11]. For practical purposes it might be useful to have a parameterization of the entropy in terms of the normalized fluctuations, which is physically transparent since entropy and fluctuations are strongly correlated [11]:

$$\frac{S}{N} = -41.68 + \frac{41.68}{(1-\frac{\langle(\Delta N)^2\rangle}{\bar{N}})^{0.022}} + 2.37\frac{\langle(\Delta N)^2\rangle}{\bar{N}} - 0.83(\frac{\langle(\Delta N)^2\rangle}{\bar{N}})^2 \quad (6)$$

For bosons, in the same scenario, we use a Bose-Einstein distribution n(p) for the particles. But we need to consider the cases above or below the critical temperature $T_c = \frac{2\pi}{[2.612(2s+1)]^{2/3}}\frac{\hbar^2}{m}\rho^{2/3}$ at a given density ρ with spin s, we obtain [11]:

$$\sigma_{xy}^2 = \bar{N}(2mT)^2\frac{g_{\frac{7}{2}}(1)}{g_{\frac{3}{2}}(1)} \qquad (T < T_c) \quad (7)$$

$$\sigma_{xy}^2 = \bar{N}(2mT)^2\frac{g_{\frac{7}{2}}(z)}{g_{\frac{3}{2}}(z)} \qquad (T > T_c) \quad (8)$$

where the $g_n(z)$ functions are well studied in the literature [11] and $z = e^{\mu/T}$ is the fugacity which depends on the critical temperature for Bose condensate and thus on the density of the system and the chemical potential μ [11]. The quadrupole fluctuations depend on temperature and density through T_c, therefore we need more information in order to be able to determine both quantities for $T > T_c$.

We can calculate the multiplicity fluctuations of d, α etc in the same framework again. Fluctuations are larger than the average value and diverge near the critical point. We will show the results later. Interactions and finite size effects will of course smooth the divergence [4]. These results are very important and could be used to pin down a Bose condensate.

Two solutions are possible depending whether the system is above or below the critical temperature for a Bose condensate. Below the critical point, Eq. (7) can be used to calculate T and then multiplicity fluctuation gives the critical temperature and the corresponding density. Above the critical point it is better to estimate the chemical potential first and then derive the temperature, critical temperature and density. The details can be seen in coming paper.

The CoMD-α Model has been developed to study collisions between alpha cluster candidates and to search for Bose condensates in alpha-clustered nuclear systems. In CoMD-α case, the nuclear ground states have an alpha-clustered structure. The boson nature of alpha-clusters is taken into count in the two-body collision term by means of the Bose-Einstein blocking factor $\Pi = (1 + \overline{f_1})(1 + \overline{f_2})$, where $\overline{f_i}$ is the average occupasion probability for alpha i=1, 2. We calculate this factor Π before the collision and Π' after the collision. If $\Pi' > \mathit{\Pi}$, the collision will be accepted, otherwise, the collision will be rejected. We will look at the multiplicity fluctuations for alpha particles. Later, we can select the particular events from experimental data which only have alpha like mass fragments. Then we can compare the model data with experimental data.

To illustrate the strength of our approach we have performed calculations for the system $^{40}Ca + ^{40}Ca$ at fixed impact parameter b=1fm and beam energies E_{lab}/A ranging from 4 MeV/A up to 100 MeV/A. Collisions were followed up to a maximum time t=1000 fm/c in order to accumulate enough statistics. A complete discussion of these simulations can be found in [6], here we will use the results to compare the different approximations.

In Fig. 1 we plot the temperature versus density as obtained from the quadrupole and multiplicity fluctuations. The top panel refers to protons while the bottom to neutrons. As we can see from the figure, the results obtained using the fit functions, Eqs.(2) and (3), deviate slightly from the lowest order approximations [6]. This is a signature that we are in the fully quantum regime for the events considered. For comparison, in the same plot we display the classical temperatures which are systematically higher than the quantum ones[10].

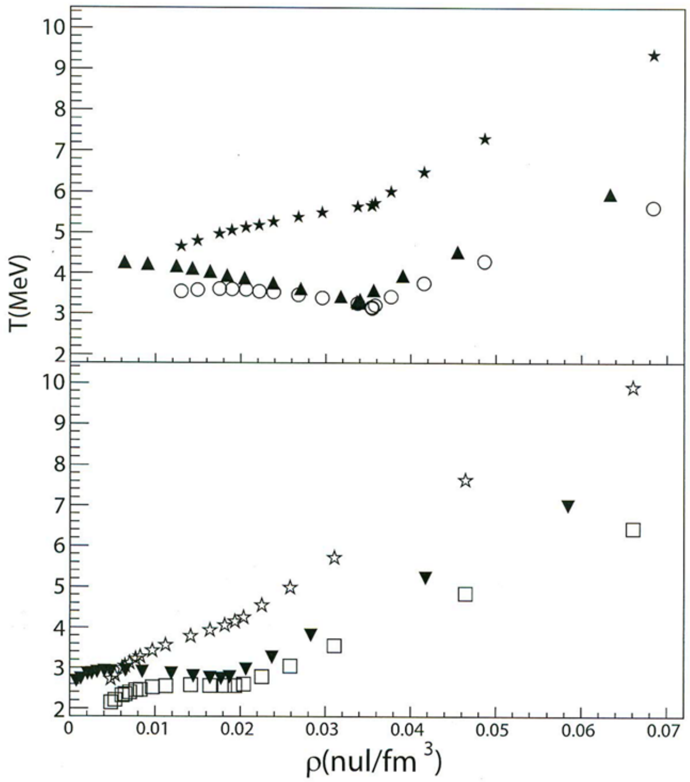

FIG. 1. Temperatures versus density normalized to the ground state density $\rho_0 = 0.16\text{fm}^{-3}$, derived from quantum fluctuations, Eqs. (2, 3). Open dots and open squares are the approximation at the lowest order in $\frac{T}{\varepsilon_f}$, full stars and open stars are the classical cases similar to [10], the full triangles are the numerical results. The top panel refers to protons and the bottom panel refers to neutrons.

To better summarize the results we plot in Fig. 2 (top panel), the energy density $\varepsilon = \langle\frac{E_{th}}{A}\rangle\,\rho$ versus temperature [6]. Different particle types scale especially at high T where Coulomb effects are expected to be small. A rapid variation of the energy density is observed around $T{\sim}2\text{MeV}$ for neutrons and $T{\sim}3\text{MeV}$ for protons which indicates a first order phase transition, or a crossover. As we see from the figure, the numerical solution of the Fermi integrals gives small corrections while keeping the relevant features obtained in the lowest approximation intact. This again suggests that in the simulations the system is fully quantal. We also notice that Coulomb effects become negligible at $T \gg 3\text{MeV}$ where the phase transition occurs. The smaller role of the Coulomb field in the phase transition has recently been discussed experimentally in the framework of the Landau's description of phase transitions [12].

In order to confirm the origin of the phase transition, it is useful to derive the entropy density $\Sigma = \langle\frac{S}{N}\rangle\,\rho$ which is plotted in the bottom panel of Fig. 2. The rapid increase of the entropy per unit volume is due to the sudden increase of the number of degrees of freedom (fragments) with increasing T. The entropy can be also derived using the law of mass action from the ratio of the produced number of deuterons to protons (or neutrons) $R_{d,p(n)}$ [4, 13]:

$$\frac{S}{N}\Big|_{d/p(n)} = 3.95 - \ln R_{d/p(n)} - 1.25\frac{R_{d/p(n)}}{1+R_{d/p(n)}} \quad (9)$$

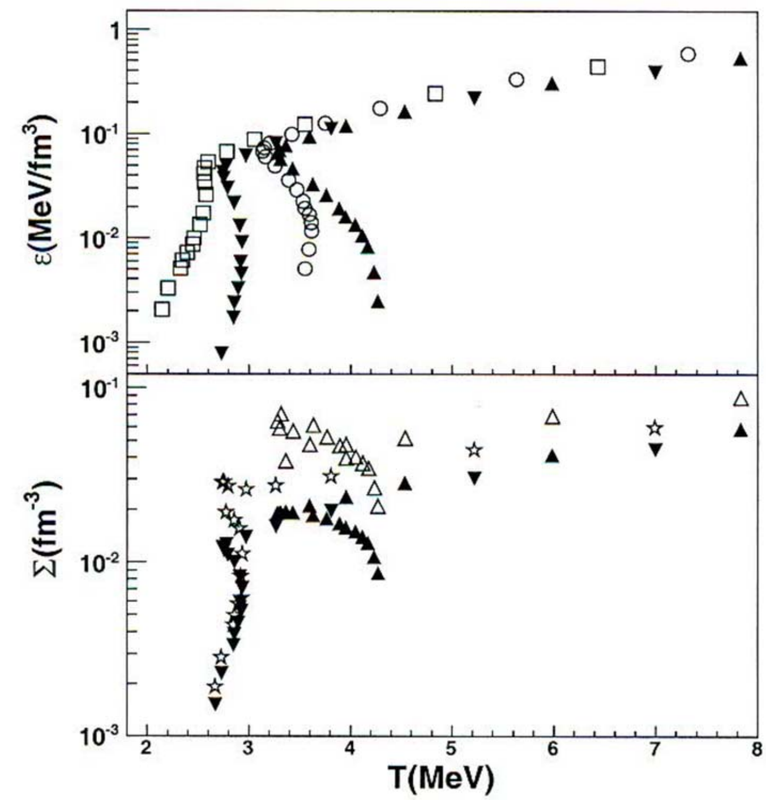

FIG. 2. (Top) Energy density versus temperature. Symbols as in Fig.1; (Bottom) entropy density versus temperature. The opens symbols refer to the entropy density calculated from the ratios of the produced number of deuterons to protons (triangles) (neutrons-stars), Eq. (9).

We find an overall qualitative good agreement of the entropy density to the quantum results, especially for neutrons. Very interesting is the good agreement for neutrons at low T where the particles are emitted from the surface of the nuclei which are at low density, see also Fig.1. Such a feature is not present for the protons due to larger Coulomb distortions. There is a region near the transition ($T \sim 3$MeV), where both ratios do not reproduce the quantum results. However, at large temperatures it seems that all methods converge as expected. Recall that this method is valid only if d, n and p are produced in the reactions. If different fragments are produced then the entropy derivation should be modified to include more complex nuclei [4]. The CoMD model favors the formation of deuterium since it is over bound. We expect data to be quite different since alpha production is important but qualitatively we expect a similar behavior as discussed here.

In Fig 3, we plot the multiplicity fluctuation versus reduced temperature $t = \frac{T-T_c}{T_c}$. The multiplicity fluctuations diverge at the critical temperature where t=0. One can calculate the multiplicity fluctuation numerically above the critical temperature. Below the critical temperature, one can obtain the behavior of the multiplicity fluctuation according to Landau's theory. It is driven by the interaction but doesn't depend on the detail of interaction.

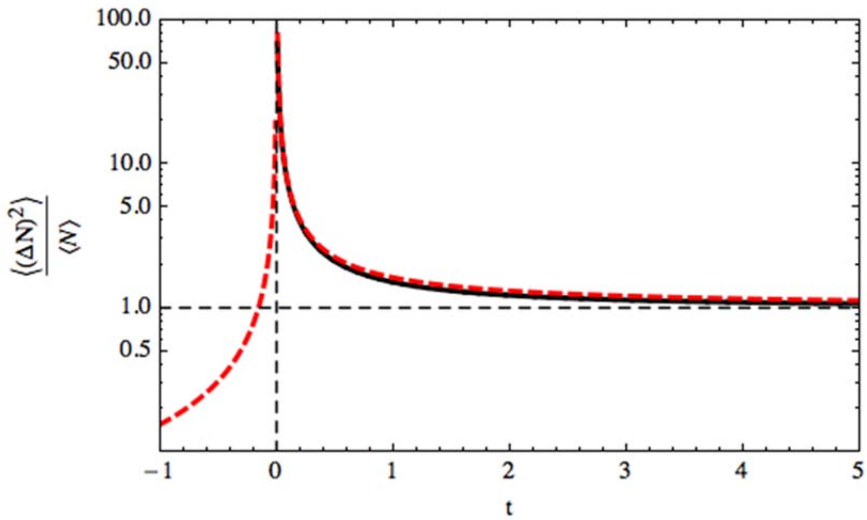

FIG. 3. Multiplicity fluctuation versus reduced temperature t.

In Fig 4, we plot the multiplicity fluctuations for alpha particle versus E_{cm}/A. As one can see, the

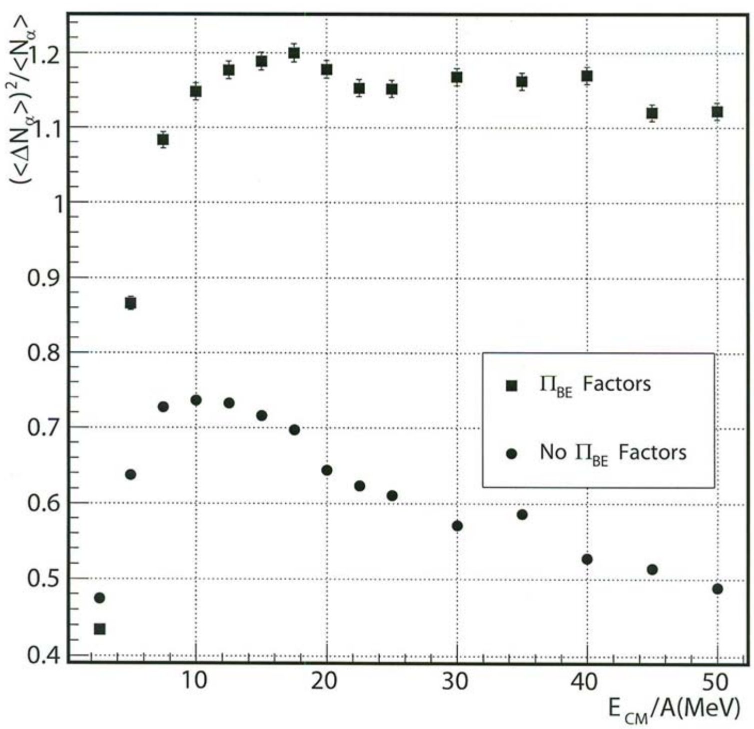

FIG. 4. Multiplicity fluctuation versus energy per nucleon in center of mass in CoMD-α. The square is with Bose-Einstein blocking factor and the circle is without Bose-Einstein blocking factor.

multiplicity fluctuations of alpha particle are systematically larger in the Bose-Einstein blocking case. This means the multiplicity fluctuations can be used as a probe to study Bose condensate phenomena in nuclear physics. Of course, the interactions and finite size effects will smooth the divergence, this may be the reason we don't see the divergence in the model.

In conclusion, in this work we have addressed a general method for deriving densities and temperatures of weakly interacting fermions and bosons. For high temperatures and small densities the classical result is recovered as expected. We have shown in CoMD calculations that the effect of higher order terms give small differences in the physical observables considered in this paper but they could become large when approaching the classical limit. To overcome this problem we have produced suitable parameterizations of quadrupole and multiplicity fluctuations which are valid at all temperatures and densities. The results obtained in this paper are quite general and they could be applied to other systems. The CoMD-αresults show that the multiplicity fluctuations are a good probe to investigate boson condensate.

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